From Identical Particles to Frictionless Flow

John Chalker

Superfluids and superconductors

Liquid helium

Electrons in (some) metals

Dilute atomic gases

Discovery of Superconductivity

H.K. Onnes

1908

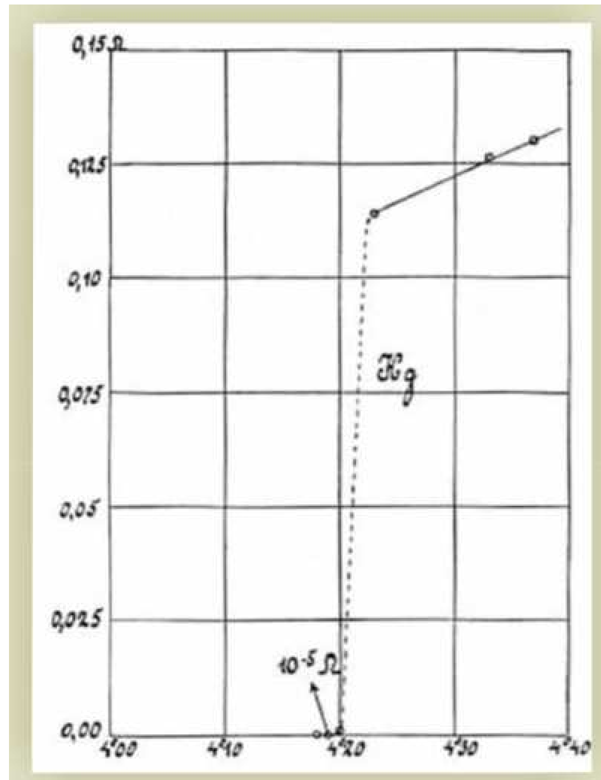
Liquification of He

1911

Observation of superconductivity in Hg

1913

Nobel Prize in Physics



“During an hour the current was observed not to decrease appreciably”

Not Spotting Superfluidity

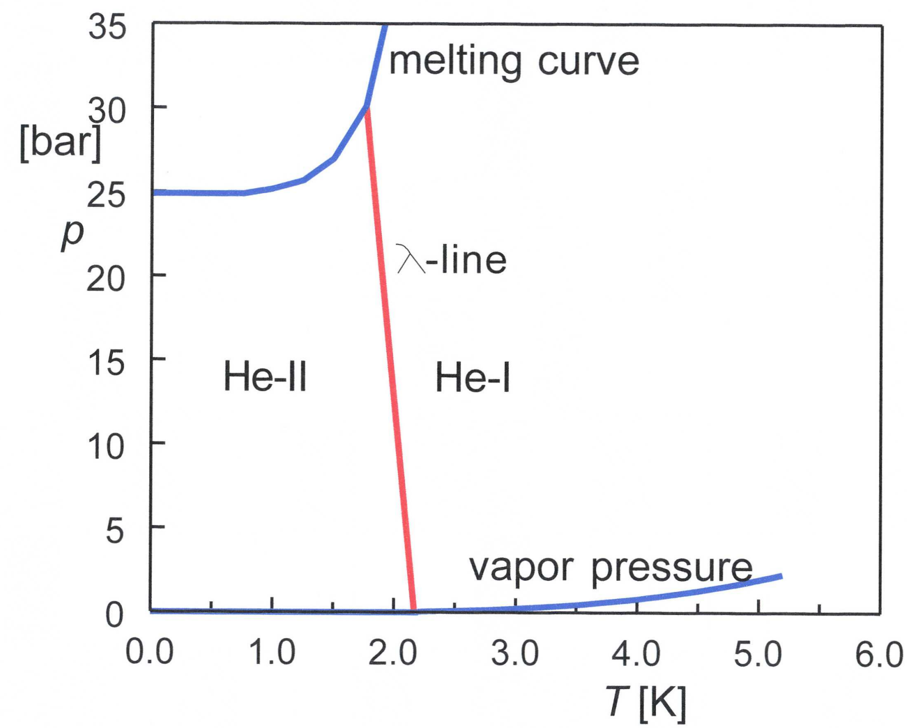
H.K. Onnes

1908

Liquification of He

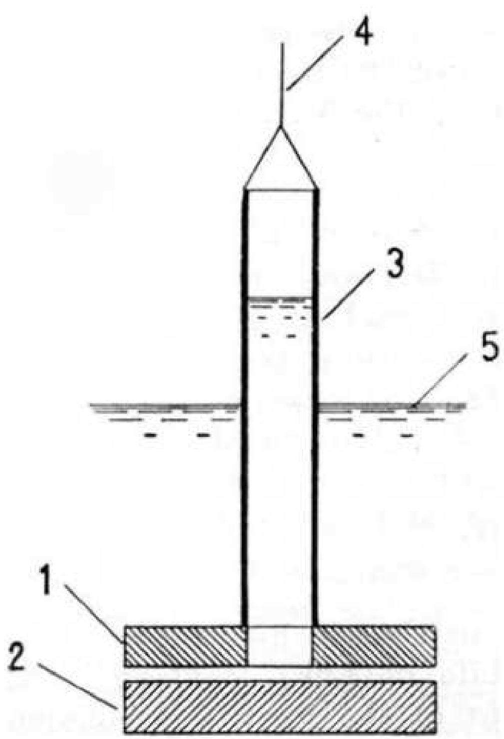
1911

Change in He at $\sim$ 2K



Discovery of Superfluidity

P. Kapitza 1937 Nobel Prize in Physics 1978



NATURE JAN. 8, 1938

"From the measurements we can conclude that the viscosity of helium II is at least 1,500 times smaller than that of helium I . . . . The present limit* is perhaps sufficient to suggest, by analogy with supraconductors, that the helium . . . enters a special state that might be called a 'superfluid'."

P. KAPITZA.

Institute for Physical Problems,
Academy of Sciences,
Moscow.
Dec. 3.

* **By 1965: viscosity $\leq 10^{11}\times$ smaller**

Quantum basics

Wavefuntion $\Psi(\mathbf{r}) \equiv |\Psi(\mathbf{r})| e^{i\varphi(\mathbf{r})}$

probability density $|\Psi(\mathbf{r})|^2$

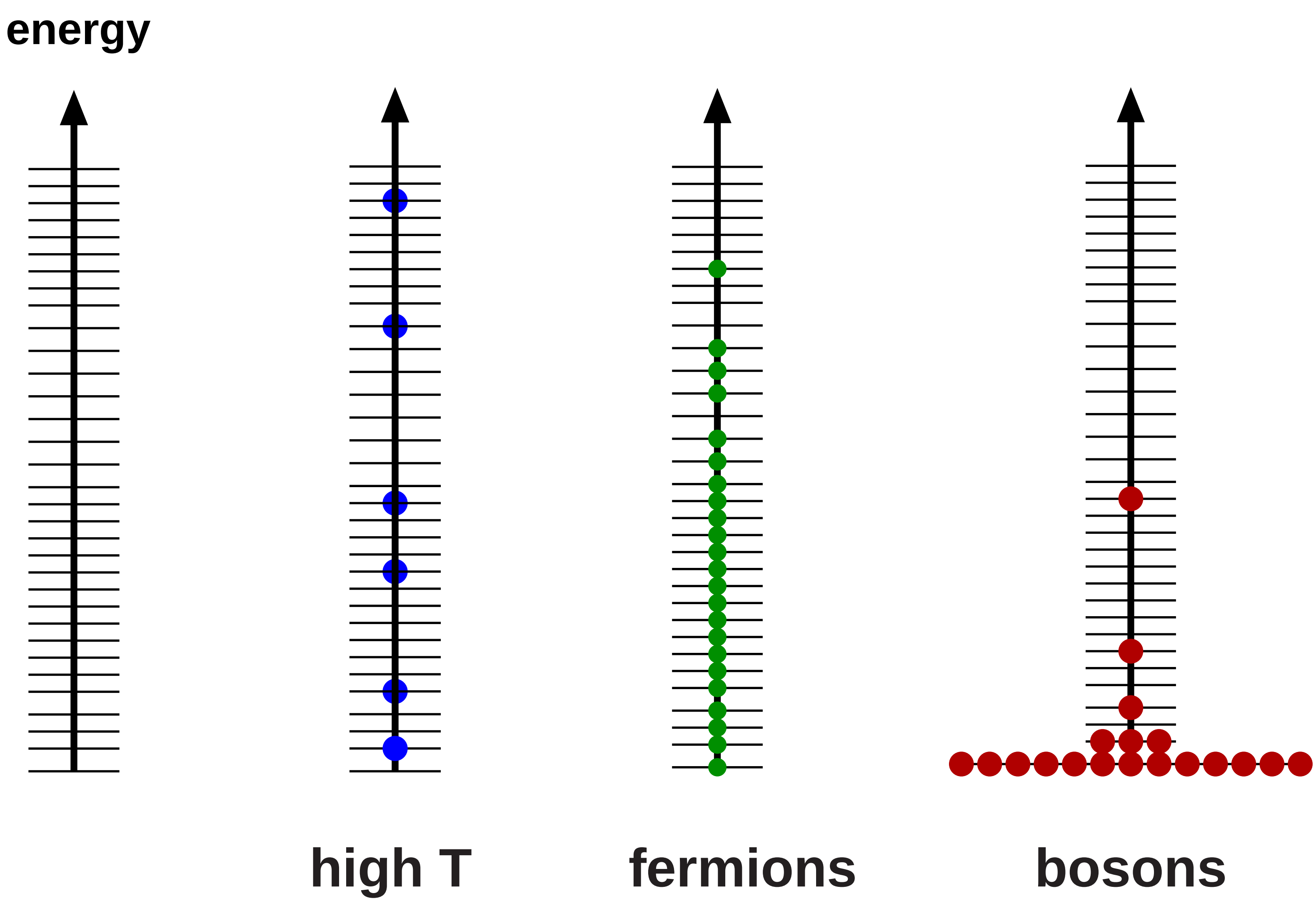
de Broglie: $\Psi(\mathbf{r}) \propto e^{i\mathbf{k}\cdot\mathbf{r}}$ **for momentum** $\hbar\mathbf{k}$

Slow particles have long wavelengths

high temperature: **wavelength** $\ll$ **particle spacing**

degeneracy temp: **wavelength** $\sim$ **particle spacing**

Quantum statistics



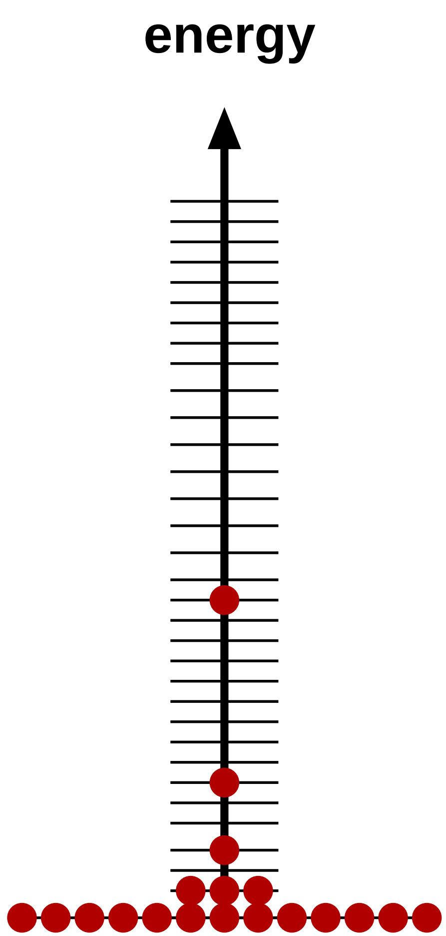
Quantum statistics

	Bosons	Fermions
Liquid helium	^{4}He	^{3}He
Electrons in metals		
Dilute atomic gases	^{87}Rb ^{23}Na	^{40}K

Quantum statistics

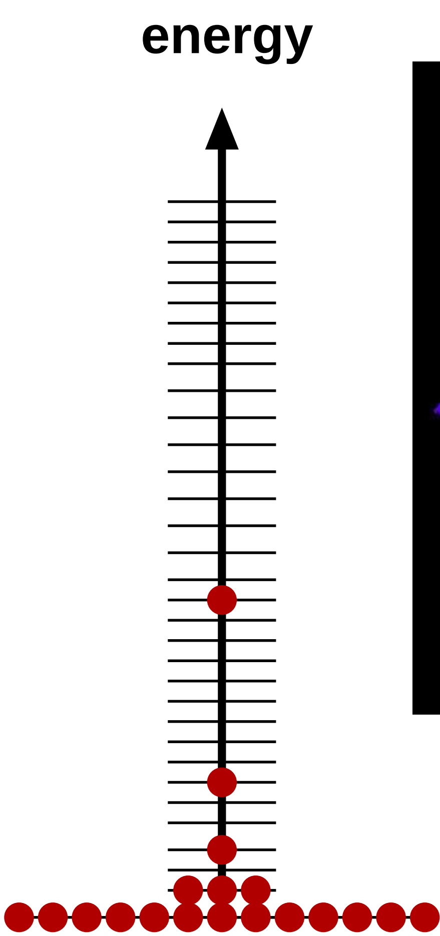
	Bosons	Fermions
Liquid helium	^{4}He	^{3}He
Superfluid transition temperature	2.17 K	0.00249 K

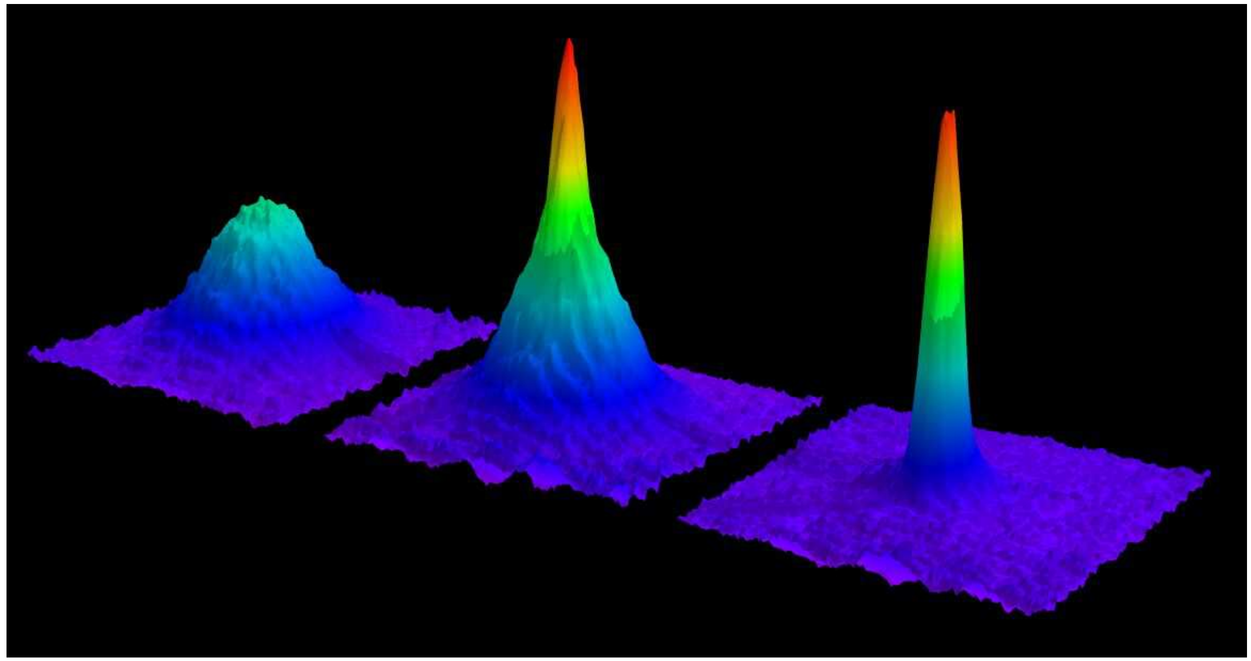
Bose-Einstein condensation



F. London (1938)

Bose-Einstein condensation



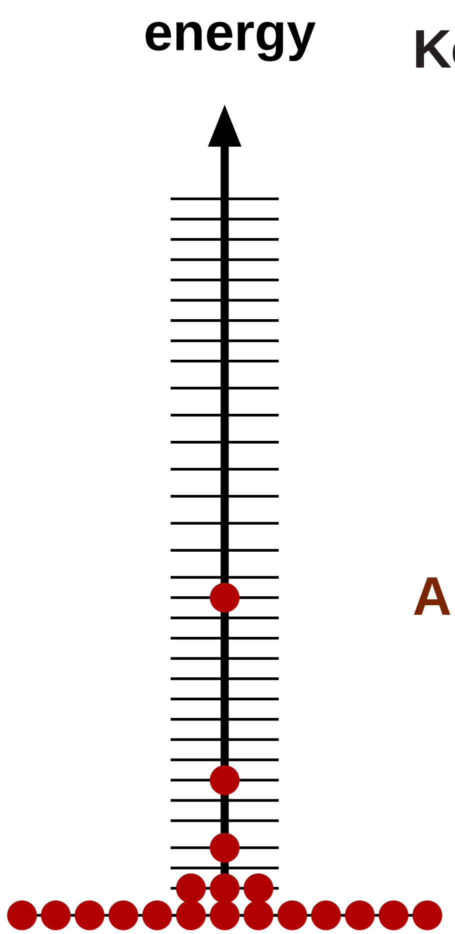


Velocity distributions vs temperature

W. Ketterle *et al* (1995)

Nobel Prize in Physics 2001: E. Cornell, W. Ketterle and C Weimann.

Superfluidity in Bose liquids



Key idea: describe whole system with

collective wavefunction $\Psi(\mathbf{r})$

$\sim$ 'local average'

of particle wavefunctions

Analogy with classical symmetry breaking

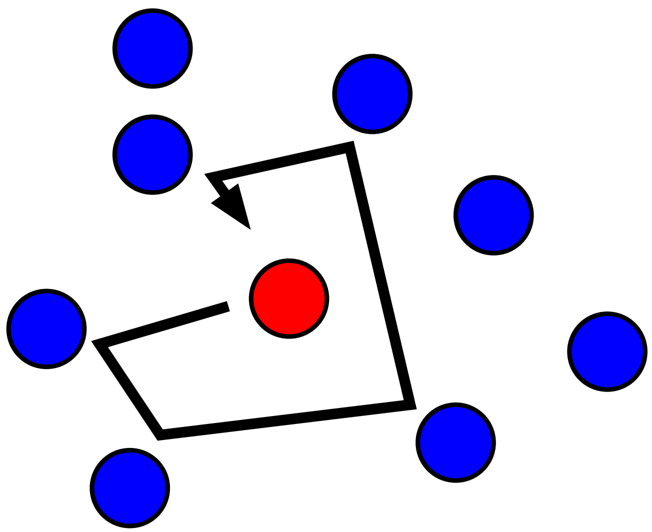
classical fluid $\leftrightarrow$ solid

'position' has macroscopic meaning

only in solid

Excitations in superfluids

Give energy to single atom?



Requires **quantum wavelength $\ll$ interatomic space**

– high energy, so excluded at low temperature

Excitations in superfluids

Share energy between many atoms?

Use collective wavefunction $\Psi(\mathbf{r},t)$ to describe excitation

What's the dynamics of $\Psi(\mathbf{r},t)$?

Start from Schrödinger eqn

$$i\hbar\frac{\mathrm{d}\Psi(\mathbf{r},t)}{\mathrm{d}t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$

and allow for interactions between particles

$$V(\mathbf{r}) \rightarrow V(\mathbf{r}) + \int \mathrm{d}\mathbf{r}' U(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}')|^2$$

Excitations in superfluids

Share energy between many atoms?

Use collective wavefunction $\Psi(\mathbf{r},t)$ to describe excitation

What's the dynamics of $\Psi(\mathbf{r},t)$?

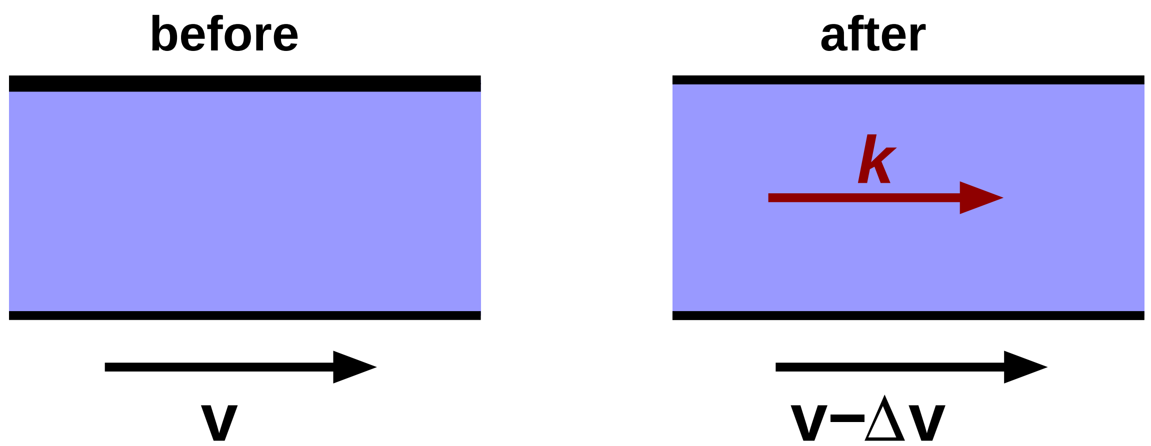
Outcome:

- **Excitations are small-amplitude waves in $\Psi(\mathbf{r},t)$**
- **Excitation energy $\varepsilon(k)$ vs wavevector k**

Viscosity from excitations?

Consider flow in rest frame of fluid

Can pipe transfer momentum to fluid by making excitation?



Conservation laws

Energy: $\frac{1}{2}M\mathbf{v}^2 = \frac{1}{2}M(\mathbf{v}-\Delta\mathbf{v})^2 + \hbar\omega(k)$ **so** $M\mathbf{v}\cdot\Delta\mathbf{v} = \hbar\omega(k)$

Momentum: $M\mathbf{v} = M(\mathbf{v}-\Delta\mathbf{v}) + \hbar\mathbf{k}$ **so** $M\Delta\mathbf{v} = \hbar\mathbf{k}$

Together $\mathbf{v}\cdot\mathbf{k} = \omega(k)$ **or** $\boxed{|\mathbf{v}| \geq \omega(k)/k}$

Landau critical velocity

Dissipation requires $|\mathbf{v}| \geq \omega(k)/k$ **Is this possible?**

L. D. Landau (1941) **Nobel Prize in Physics 1962**

Landau critical velocity

Dissipation requires $|\mathbf{v}| \geq \omega(k)/k$ **Is this possible?**

Free particle dispersion

$$\hbar\omega(k) = \frac{\hbar^2 k^2}{2m}$$

— yes!

Sound wave dispersion

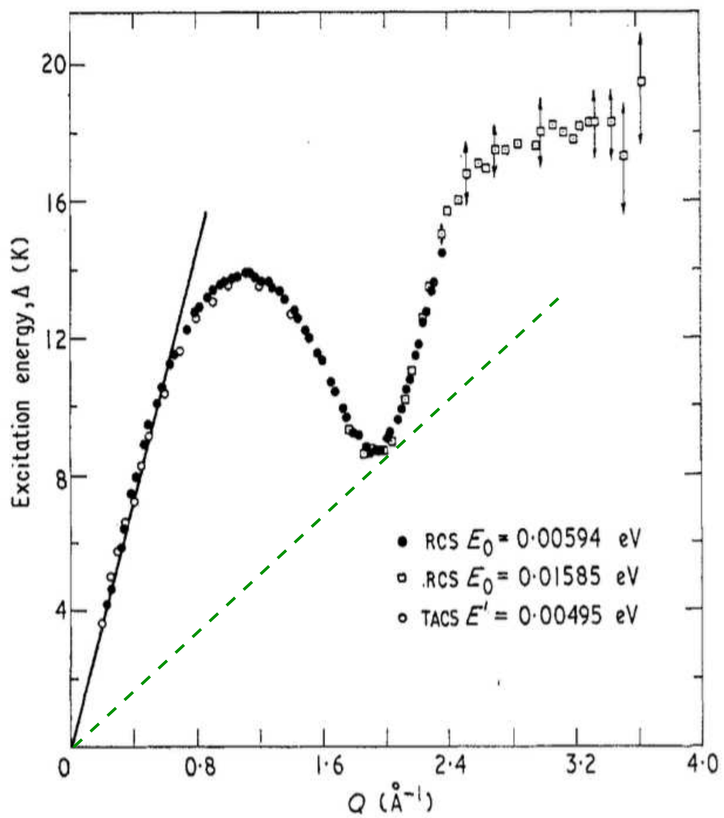
$$\omega(k) = ck$$

— for slow flow ($v < c$**) no!**

Landau critical velocity

Dissipation requires $|\mathbf{v}| \geq \omega(k)/k$ **Is condition satisfied?**

Dispersion from experiment



No dissipation below critical velocity

$\hbar\omega(k)$ **vs** k

Cowley and Woods (1971)

Superfluid flow

Key idea: describe whole system with wavefunction $\Psi(\mathbf{r})$

Interpret $|\Psi(\mathbf{r})|^2$ as superfluid density

Meaning of phase $\varphi(\mathbf{r})$ of $\Psi(\mathbf{r})$?

Recall de Broglie:

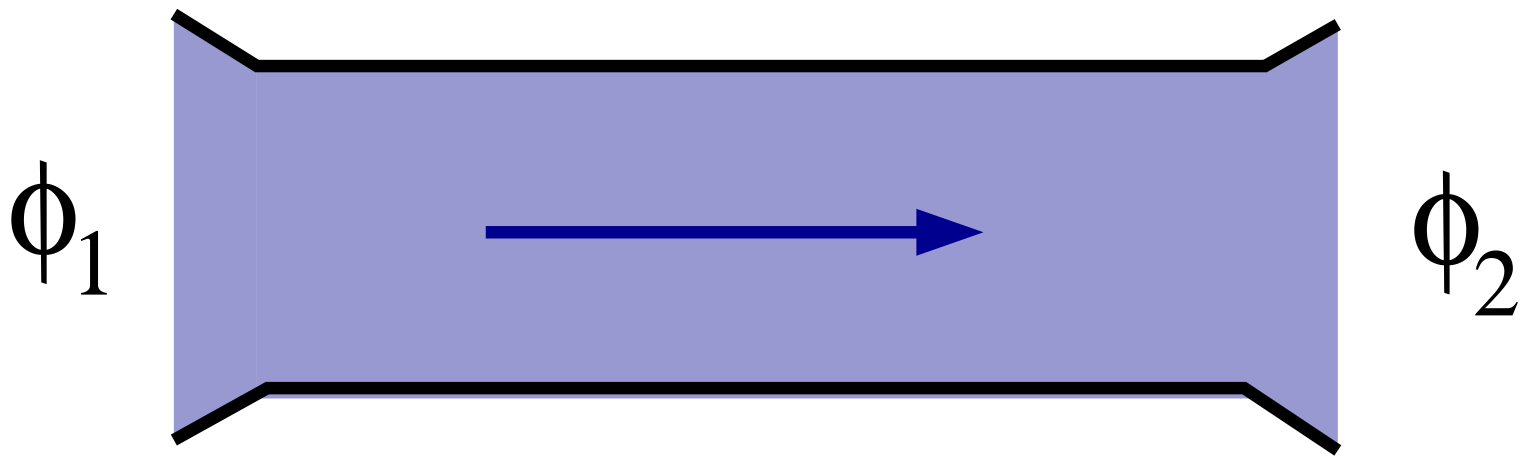
$\Psi(\mathbf{r}) \propto e^{i\mathbf{k}\cdot\mathbf{r}}$ **for momentum** $\hbar\mathbf{k}$

Superfluid velocity $\boxed{\mathbf{v} = \frac{\hbar}{m}\nabla\varphi(\mathbf{r})}$

Flow in a superfluid

Superfluid velocity $\mathbf{v} = \frac{\hbar}{m}\nabla\varphi(\mathbf{r})$

Flow in a channel

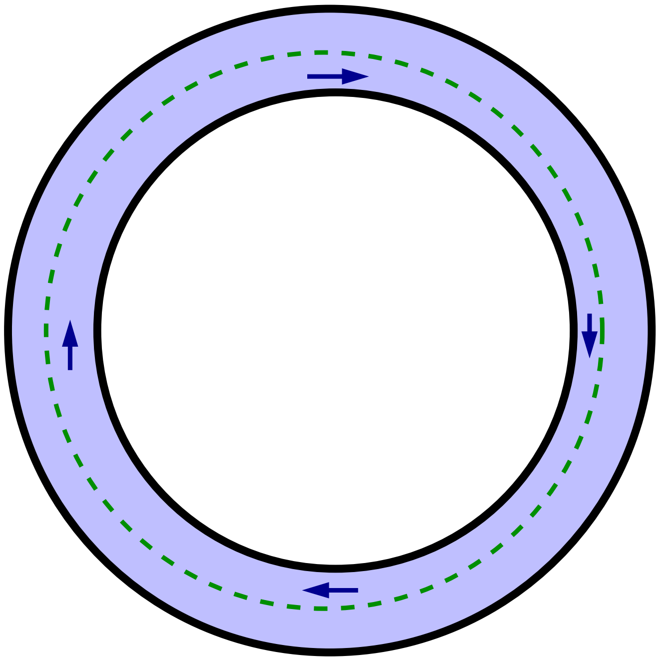


– superfluid phase gradient along length

Flow in a superfluid

Superfluid velocity $\mathbf{v} = \frac{\hbar}{m} \nabla \varphi(\mathbf{r})$

Flow around annulus



– superfluid phase gradient around circumference

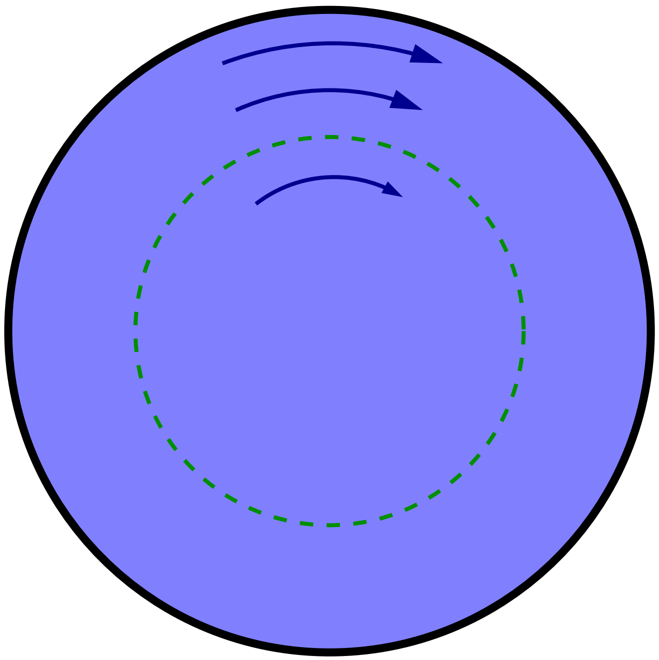
$\oint \nabla \varphi(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = 2\pi \times$ **integer**

$\Rightarrow$ **quantised circulation!**

Flow in a superfluid

Superfluid velocity $\mathbf{v} = \frac{\hbar}{m}\nabla\varphi(\mathbf{r})$

Superfluid in a rotating bucket



Rotation at fixed angular velocity impossible

$\oint \nabla\varphi(\mathbf{r}) \cdot \mathrm{d}\mathbf{r} = 2\pi \times$ **integer**

Flow in a superfluid

Superfluid velocity $\mathbf{v} = \frac{\hbar}{m} \nabla \varphi(\mathbf{r})$

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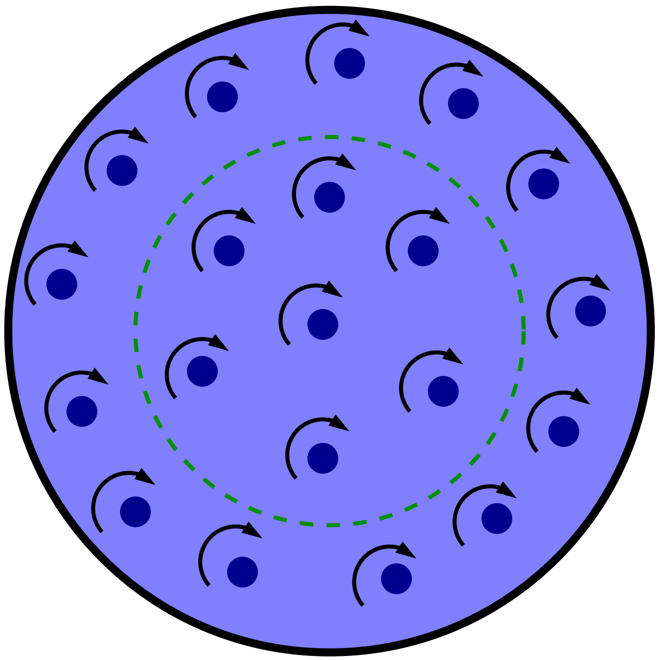


Rotation at fixed

angular velocity impossible

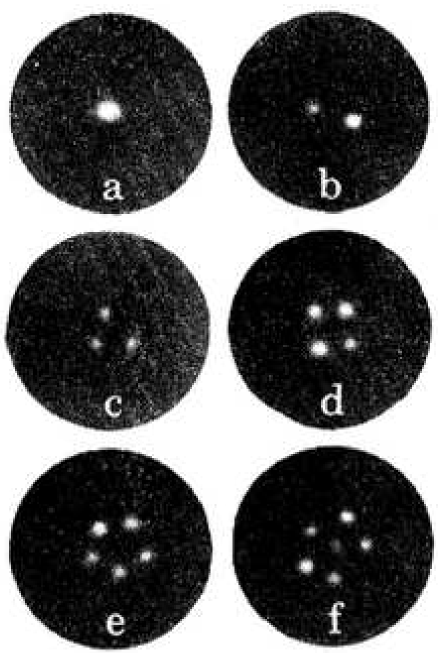
Rotating superfluid threaded

by quantised vortices

Flow in a superfluid

Superfluid velocity $\mathbf{v} = \frac{\hbar}{m}\nabla\varphi(\mathbf{r})$

Superfluid in a rotating bucket



Rotation at fixed angular velocity impossible

Rotating superfluid threaded by quantised vortices

Yarmchuk, Gordon and Packard (1979)

Container: 2 mm. Rotation rate: $\sim$ 0.5 s^{-1}

Summary

Coherence of many atoms

$\Psi(\mathbf{r})$ **becomes macroscopic variable**

Dissipationless flow

Quantised vortices